Neutrino Mixing and Big Bang Nucleosynthesis

The Universe's Lepton Number

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Outline

- Significance of large lepton number
- Constraints on relic neutrino asymmetries
  - Big Bang Nucleosynthesis
- Combining neutrino oscillations with Big Bang Nucleosynthesis

Confirmation of the large mixing angle (LMA) solution to the solar neutrino problem → Strongest constraint on the Universe’s lepton number

Baryon & Lepton Asymmetries

Baryon asymmetry: $B = \frac{n_b - n_{\bar{b}}}{n_\gamma} \sim 10^{-10}$

Lepton asymmetry: → only very weak constraints.

Charge neutrality of the universe prevents a large asymmetry in the charged leptons, but a large lepton number could reside in the neutrino sector.

How would we know?

Theoretical predictions:

- L is related to B (e.g. Lepto/Baryogenesis scenarios in which the two are connected via spheralon transitions which freeze out near the electroweak phase transition). $L \sim 10^{-10}$

- L and B are unrelated if EW symmetry was never restored.
  (and large L can prevent EW sym from being restored – Linde)

- L is much bigger than B, e.g. Afleck-Dine baryogenesis scenarios in SUSY models.

- Large L is generated at temperatures below the EW scale , eg via active-sterile neutrino oscillations. $L \sim 0.1$

- Etc, etc..

Relic neutrino background

In thermal equilibrium, the neutrinos will have Fermi-Dirac distributions:

$$f(p,\xi) = \frac{1}{1+\exp(p/T-\xi)}$$

Lepton asymmetries imply chemical potentials: ξ_ν

$e^+ + e^- \leftrightarrow \nu + \bar{\nu}$ maintains chemical equilibrium such that: $\xi_\nu = -\xi_{\bar{\nu}}$

Lepton asymmetries: $$L_\alpha = \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma} = \frac{\pi^2}{12\varsigma(3)}\left(\xi_\alpha + \frac{\xi_\alpha^3}{\pi^2}\right)$$

Such degeneracies increase the effective number of species in equilibrium:

$$\Delta N_\nu = \frac{30}{7}\left(\frac{\xi}{\pi}\right)^2 + \frac{15}{7}\left(\frac{\xi}{\pi}\right)^4$$

Unlike the CMB photons, the relic neutrino background has never been directly detected – so we have to infer its properties through indirect means.

Today, T_ν~2K, so : If m > 5x10^{-4} eV, the relic neutrinos are non-relativistic today.

Given the solar and atmospheric mass squared differences:

$$\sqrt{\delta m^2_{atm}} > 0.04\,\mathrm{eV}$$
$$\sqrt{\delta m^2_{solar(LMA)}} > 0.004\,\mathrm{eV}$$

at least 2 of the 3 neutrinos states are non-relativistic today.

Neutrino contribution to the matter density:

Energy density = mass x number density

$$\rho_\nu = m_\nu n_\nu$$

Large scale structure can tell us about ρ.

So, *provided we have a good understanding of n*, we can “weigh” neutrinos with cosmology

2dF + WMAP → sum of the 3 neutrino masses < 0.7 eV

This limit assumes n has the standard value with zero lepton number

Constraints on relic neutrino asymmetries…

…in the absence of neutrino mixing

BBN+CMB set weak bounds on the lepton numbers:

e.g. Hansen et al. (2001)

$$\text{Very weak bound for } \nu_{\mu,\tau}: \quad \left|\xi_{\mu,\tau}\right| < 2.6$$

$$\text{Stronger bound for } \nu_e: \quad -0.01 < \xi_e < 0.22$$

Note that BBN and CMB probe completely different epochs of the universe.

Neutrinos and Big Bang Nucleosynthesis

Temperature ~ MeV

Neutron to proton ratio set by the processes:

$$n + \nu_e \leftrightarrow p + e^-$$

$$n + e^+ \leftrightarrow p + \bar{\nu}_e$$

Practically all the neutrons eventually end up in Helium nuclei. All the leftover protons form Hydrogen.

$$n/p \approx \exp[-(m_n - m_p)/T]$$

These process “freeze out” when:

Interaction rate < Expansion rate

Expansion rate $\propto$ energy density

$$H = \frac{\dot{R}}{R} \propto \rho_{\text{radiation}}$$

expansion rate $\rightarrow$ contribution from $\gamma, e, \nu_e, \nu_\mu, \nu_\tau +$ antiparticles

If there were extra neutrinos (or any other relativistic particles)

- The universe would expand faster
- Weak interaction rates would freeze out earlier
- Larger n/p ratio and hence more Helium

Successful nucleosynthesis puts constraints on the expansion rate…and therefore tells us how many relativistic particles species were in thermal equilibrium.

ν_e directly affects neutron-proton equilibrium:

$$n + \nu_e \leftrightarrow p + e^-$$

$$n + e^+ \leftrightarrow p + \overline{\nu}_e$$

$$n / p \approx \exp\left[-(m_n - m_p)/T - \xi_e\right]$$

If there were an electron neutrino asymmetry:

eg. $\quad n(\nu_e) > n(\overline{\nu}_e)$

$\Rightarrow \quad n/p \quad \downarrow$

$\quad Y_P \quad \downarrow$

BBN (+CMB) constraint:

$$\left|\xi_{\mu,\tau}\right| < 2.6$$

$$-0.01 < \xi_e < 0.22$$

Note that the upper limits can only be obtained in tandom.
This is the degenerate BBN scenario in which the effects of the asymmetry in ν_e is compensated by faster expansion rate due to extra energy density.

Without this compensation the limit for ν_e would be:

$$\left|\xi_e\right| \lesssim 0.04$$

How do neutrino oscillations change things?

- Active-sterile mixing

→ LSND inspired mixing schemes

- Active-active mixing

→ Oscillations with the solar and atmospheric parameters.

Sterile neutrinos?

Active -sterile oscillations in the early universe:

$$\nu_{\text{active}} \leftrightarrow \nu_{\text{sterile}}$$

would thermalise the sterile neutrinos….
…and we know that BBN works more or less OK with just three neutrinos

In fact, successful BBN sets stringent bounds on active-sterile oscillation parameters:

eg. Assuming a BBN bound of $N_\nu < 3.4$

Naive constraint on $\nu_\mu \leftrightarrow \nu_s$ or $\nu_\tau \leftrightarrow \nu_s$ mixing :

$$\left|\delta m^2 (\sin^2 2\theta)^{1.6}\right| < 10^{-7} \text{ eV}^2$$

e.g. Dolgov; Enqvist, Kainulainen &Thomson.

BBN says N < 4 → Sterile neutrinos are cosmologically disfavoured if they mix significantly with active neutrinos.

ALL "3+1" and "2+2" models which accommodate LSND are problematic for BBN.

See di Bari(2001); Abazajian (2002) for recent analyses.

HOWEVER… there are ways out…

- Equilibration of the sterile is avoided if a lepton asymmetry is present
→the mixing angle is suppressed due to the refractive index

Foot & Volkas (1995)

- Some other much more exotic scenarios…
…low reheating scenarios, coherent majoron fields, etc …

> If MiniBooNE were to confirm LSND, it would be of *enormous* cosmological significance.

Active-Active Oscillations

Oscillations between active neutrino species in the Early Universe have received much less attention than active sterile oscillations because:

1. Oscillations do nothing if we have equal numbers of each flavour
But there may very well be asymmetries between the flavours.

2. Its a much harder problem.
Neutrino-neutrino forward scattering makes things non-linear resulting in highly non-trivial dynamics.

Savage, Malaney & Fuller. Kostelecky, Pantaleone & Samuel.

LMA Solar Neutrino Solution

The Large Mixing Angle solution has been confirmed as correct resolution of the solar neutrino anomaly → KamLAND & SNO

Best fit mixing parameters:

$$\delta m_0^2 \approx 4 \times 10^{-5}\ \mathrm{eV}^2$$

$$\sin^2 2\theta_0 \approx 0.8$$

Large-angle mixing could potentially equilibrate the flavours.

Savage, Malaney & Fuller (1991); Lunardini and Smirnov (2001).

Matter effects are quite significant and must be included to determine if equilibration would take place before weak freeze-out.

Dolgov et al. (2002); Abazajian, Beacom and Bell (2002); Wong (2002).

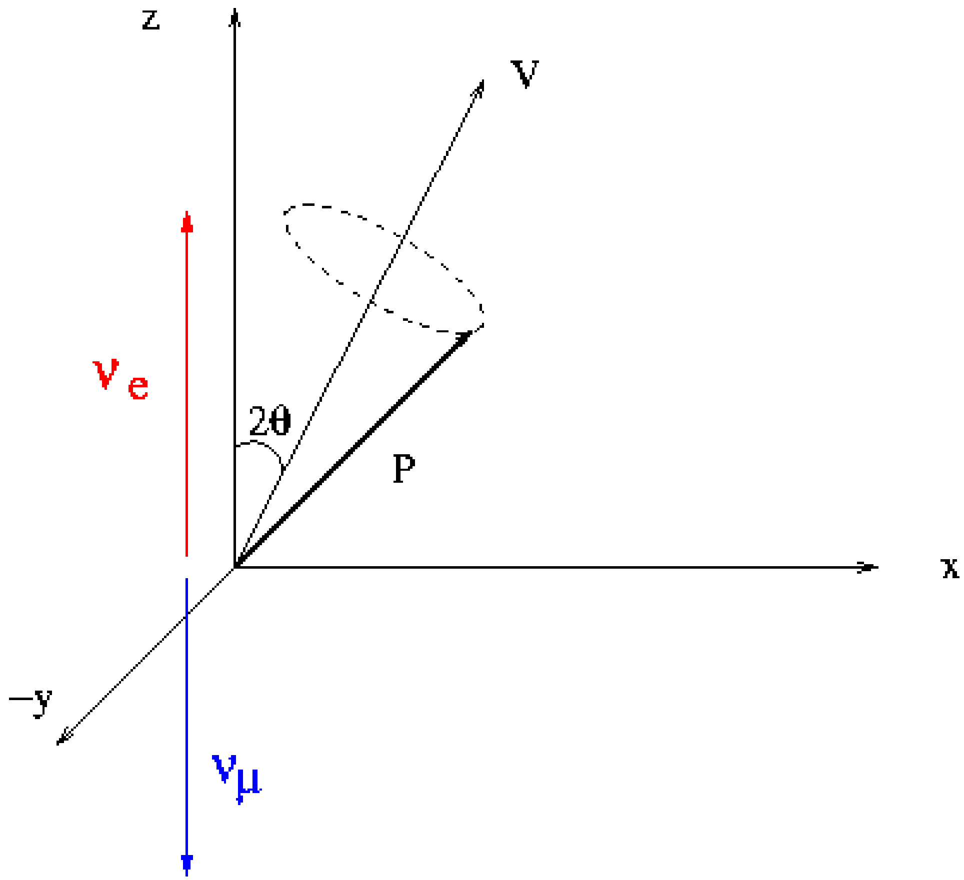
Density matrix parameterization:

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{e\mu} \\ \rho_{\mu e} & \rho_{\mu\mu} \end{pmatrix} = \frac{1}{2}\left[1 + \boldsymbol{\sigma} \bullet \mathbf{P}\right]$$

$$= \frac{1}{2}\begin{pmatrix} 1+P_z & P_x + iP_y \\ P_x - iP_y & 1-P_z \end{pmatrix}$$

“Polarisation” vector:

$$\mathbf{P} = \left(P_x, P_y, P_z\right)$$



Oscillations are described by the precession of the P vector
-- just like a spin precessing in a magnetic field.

Evolution equations:

$$\partial_t \mathbf{P}_p = \left(\mathbf{A}_p + \alpha\,\mathbf{I}\right) \times \mathbf{P}_p$$

$$\partial_t \overline{\mathbf{P}}_p = \left(-\mathbf{A}_p + \alpha\,\mathbf{I}\right) \times \overline{\mathbf{P}}_p$$

A=vacuum mixing term + non-neutrino background

I= neutrino-neutrino forward scattering term

$$\mathbf{A}_p = \frac{\delta m_0^2}{2p}\left(\sin 2\theta_0 \hat{x} - \cos 2\theta_0 \hat{z}\right) + V_B \hat{z}$$

$$\mathbf{I} = \int \frac{d^3(p/T)}{(2\pi)^3}\left[\mathbf{P}_p - \overline{\mathbf{P}}_p\right]$$

Behaviour of single mode in absence of ν–ν forward scattering term:

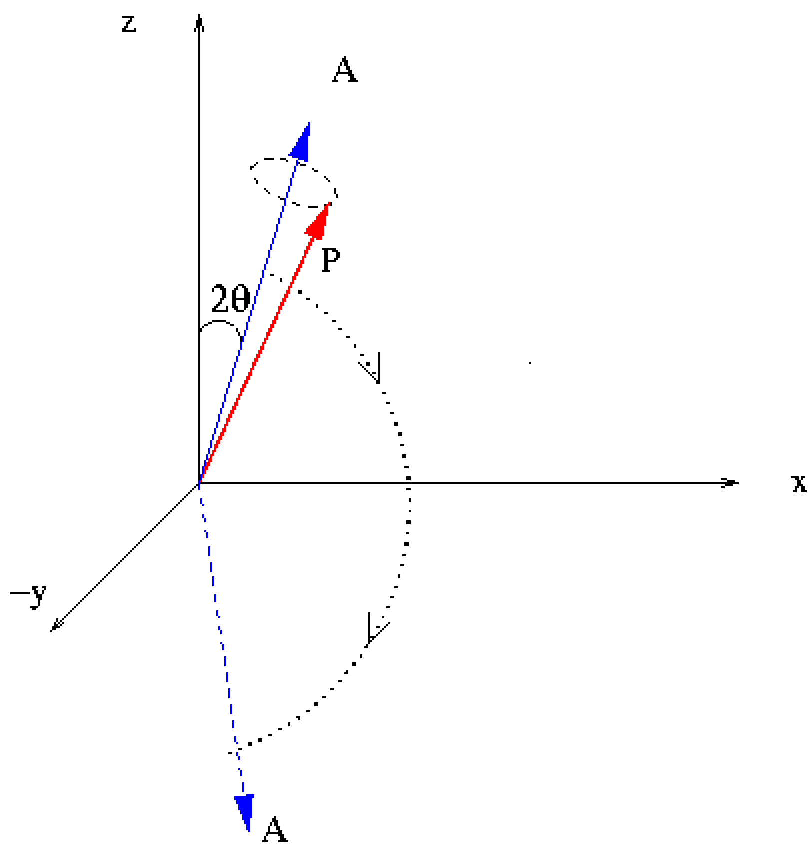
$$\partial_t \mathbf{P}_p \approx \mathbf{A}_p \times \mathbf{P}_p$$

$$\mathbf{A}_p = \frac{\delta m_0^2}{2p}\left(\sin 2\theta_0 \hat{x} - \cos 2\theta_0 \hat{z}\right) - \frac{8\sqrt{2} G_F p}{3M_W^2} E_e \hat{z}$$

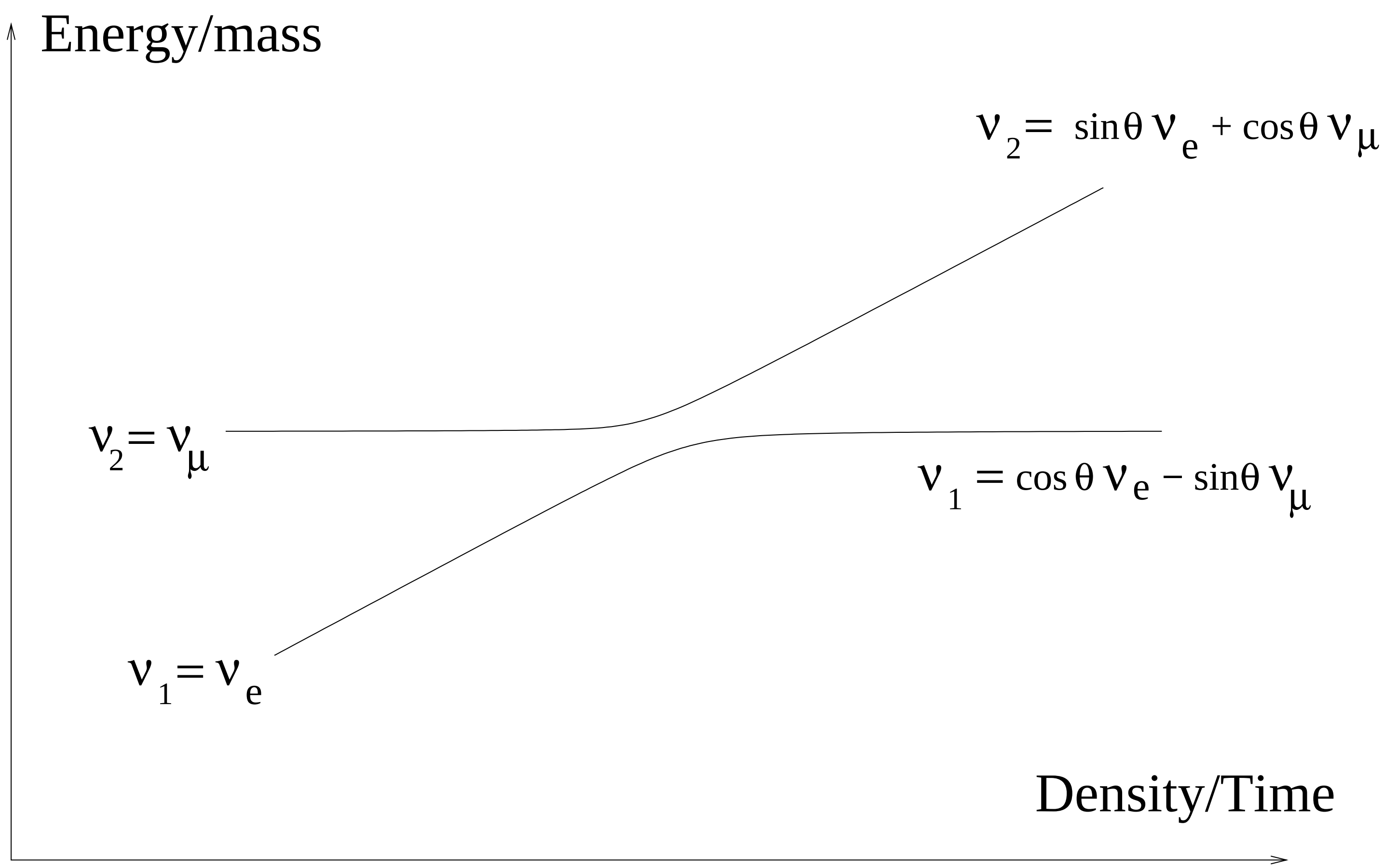
- The thermal potential is initially large and decreases slowly as the temperature falls

- A rotates from the Z-axis to a direction specified by the vacuum mixing parameters.

- The polarisation vector is initially aligned with A, and follows A as it makes this transition – this is just an adiabatic MSW transition


z
A
P
2θ
x
−y
A

MSW transitions



Savage, Malaney & Fuller (1991)

The neutrino-neutrino forward scattering term makes the problem highly non-linear.

Note that this term includes both diagonal and off-diagonal refractive indices, the off-diagonal contributions coming from forward scattering processes of the type:

$$\nu_\alpha(p) + \nu_\beta(k) \to \nu_\alpha(k) + \nu_\beta(p)$$

Pantaleone (1992).

Also, Friedland & Lunardini, in preparation.

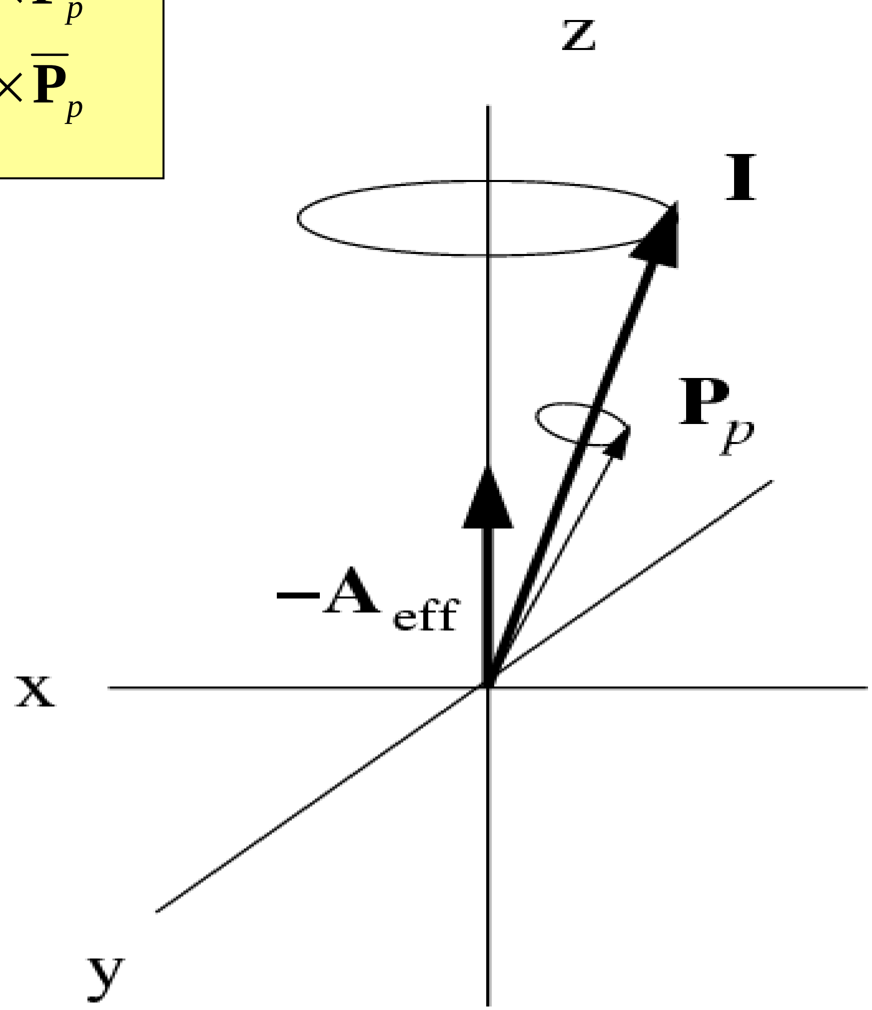
The non-linear term dominates in size as long as the initial asymmetry is larger than about:

$$L > 10^{-5}$$

…that is, it dominates for all initial asymmetries of interest.

$$\partial_t \mathbf{P}_p = \left(\mathbf{A}_p + \alpha\,\mathbf{I}\right)\times \mathbf{P}_p$$
$$\partial_t \overline{\mathbf{P}}_p = \left(-\mathbf{A}_p + \alpha\,\mathbf{I}\right)\times \overline{\mathbf{P}}_p$$

$$\mathbf{I} = \int dp(\mathbf{P}_p - \overline{\mathbf{P}}_p)$$



Synchronisation

•In the absence of neutrino-neutrino forward scattering, each momentum mode has a different oscillation frequency.

•Including ν-ν forward scattering pins all the momentum modes together so they oscillate in sync.

Kostelecky, Pantaleone and Samuel.

The polarisation vector for each momentum mode is pinned to the collective polarisation vector I…like a collection of magnetic moments.

Pastor, Raffelt and Semikoz (2002)

The evolution of the collective polarisation is determined according to:

$$\partial_t \mathbf{I} \approx \mathbf{A}_{\text{eff}} \times \mathbf{I}$$

$$\mathbf{A}_{\text{eff}} \approx \frac{1}{\mathbf{I}^2} \int \mathbf{A}_p \left(\mathbf{P}_p + \overline{\mathbf{P}}_p \right) \bullet \mathbf{I}$$

When the neutrino self potential dominates, it synchronises the ensemble so that all neutrinos behave as though they have the same effective momentum.

Parameters describing the evolution of the collective polarisation:

$$\mathbf{A}_{\text{eff}} \approx \Delta_{\text{sync}} \left(\sin 2\theta_{\text{sync}} \hat{x} - \cos 2\theta_{\text{sync}} \hat{z} \right)$$

$$\Delta_{\text{sync}} \propto \xi$$

Very sensitive to initial asymmetry.

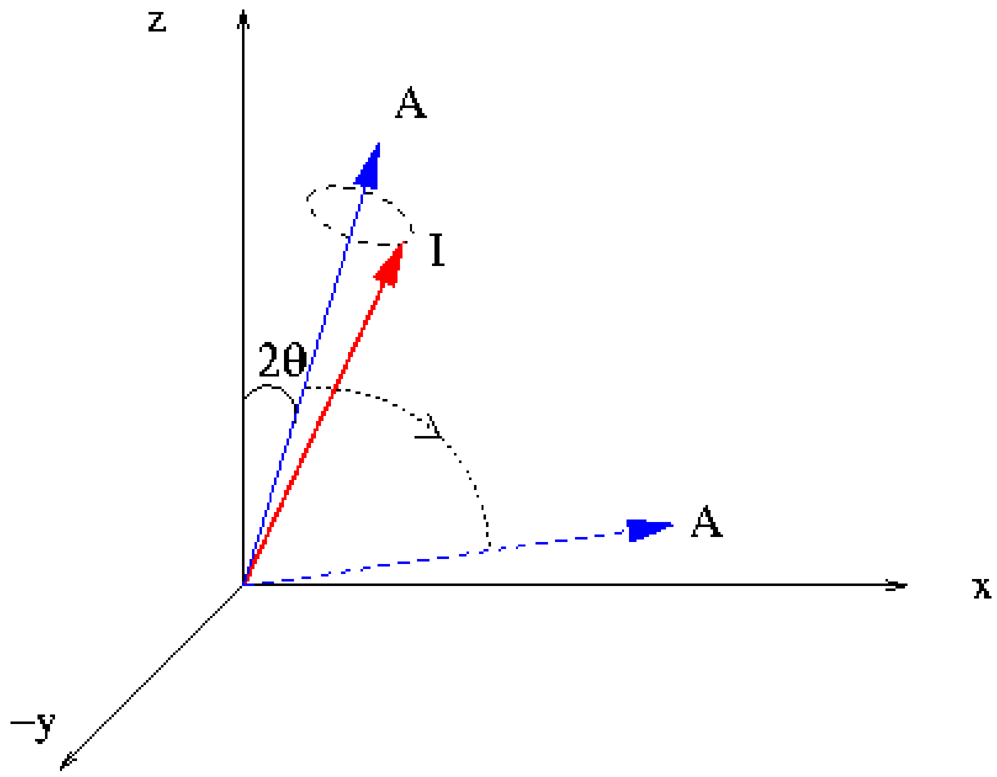
Effective mixing angle , however is insensitive to the initial asymm:

$$\sin^2 2\theta_{\text{sync}} = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + \left[\cos 2\theta_0 - V_B(p_{\text{sync}}) \right]^2}$$

$$\frac{p_{\text{sync}}}{T} = \pi \sqrt{1 + \xi^2 / 2\pi^2} \approx \pi$$

Abazajian, Beacom and Bell (2002);
Wong (2002).

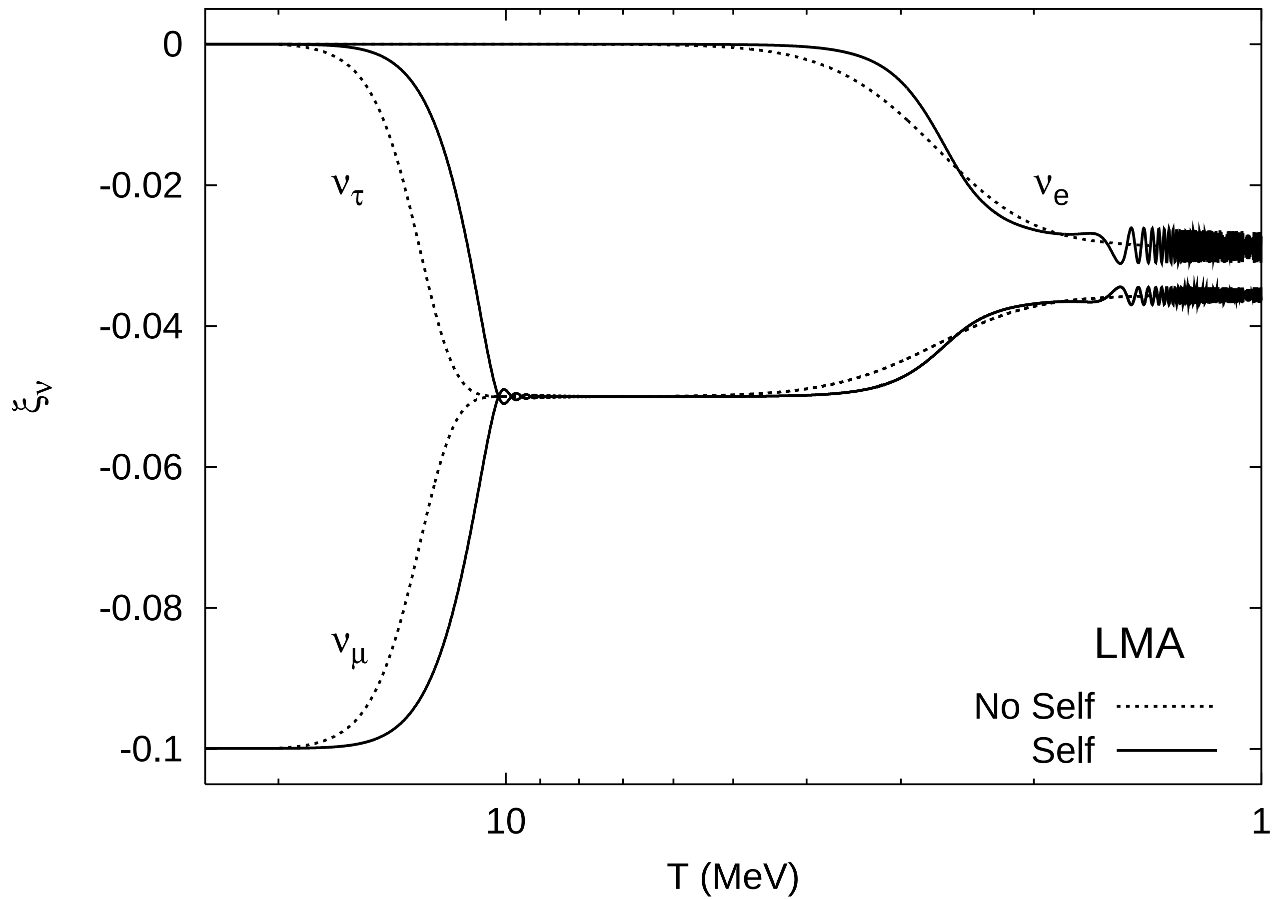
It is this “synchronised mixing angle” that determines when the flavour equilibration occurs.



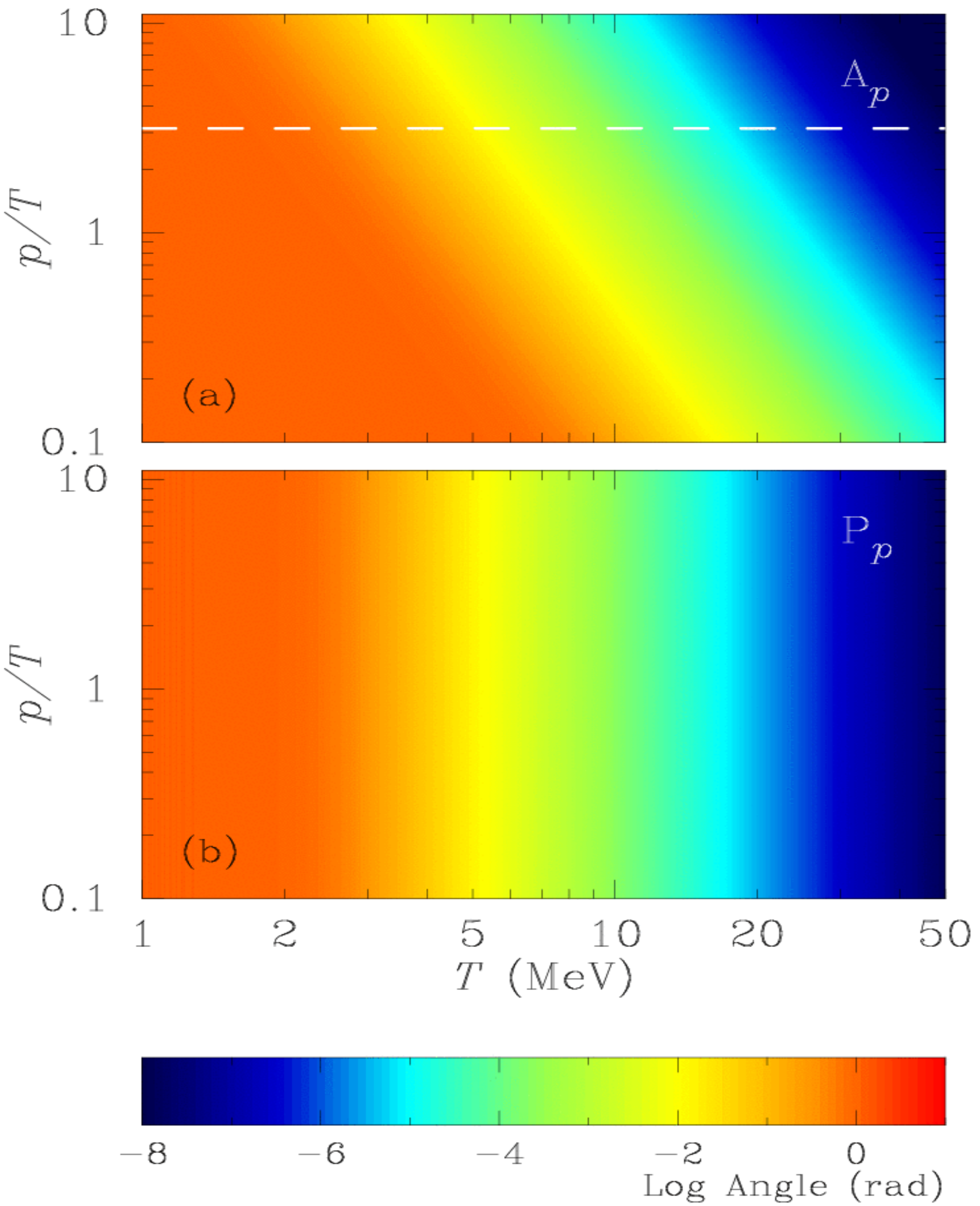
In the standard three-flavour picture of neutrino mixing:

$\nu_\mu - \nu_\tau$ equilibration takes place at T ~ 10 MeV

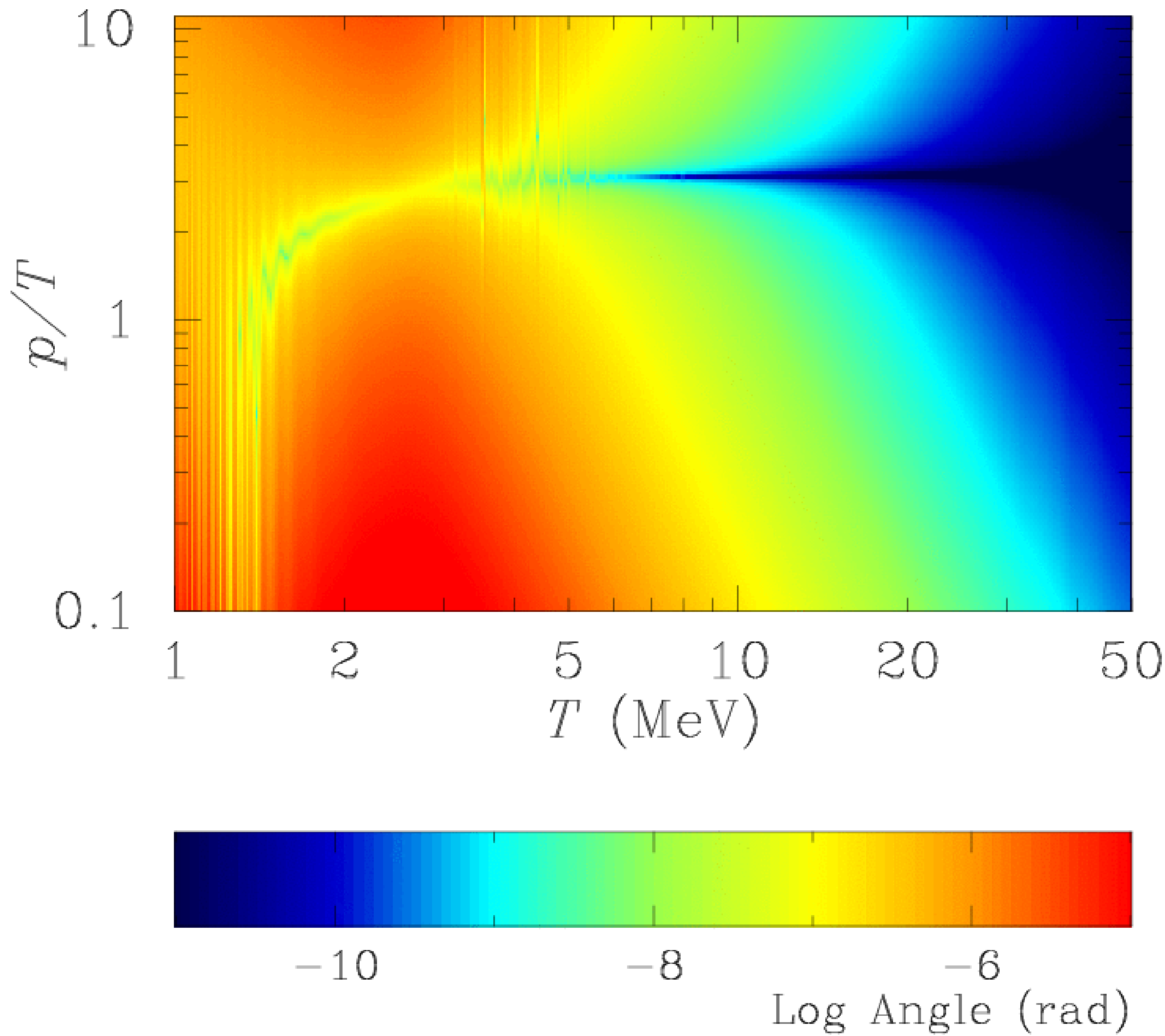
$\nu_e - \nu_\mu / \nu_\tau$ equilibration takes place at T ~ 2 MeV.



Dolgov, Hansen, Pastor, Petcov, Raffelt &Semikoz. (2002)



10
1
0.1
p/T
A_p
(a)
10
1
0.1
p/T
P_p
(b)
1
2
5
10
20
50
T (MeV)
−8
−6
−4
−2
0
Log Angle (rad)



10
1
0.1
p/T
1
2
5
10
20
50
T (MeV)
−10
−8
−6
Log Angle (rad)

New Constraints:

$$\xi_e^f \approx \left(\frac{1-\cos 2\theta_0}{2}\right)\xi_\mu^i$$

Using the best fit value of the LMA mixing angle $\sin^2 2\theta_0 \approx 0.8$

$$\xi_e^f < 0.04 \quad \Rightarrow \quad \xi_\mu^i < 0.3$$

Collisional processes will help make the equilibration more complete, as does non-zero U_{e3}.

Degenerate BBN is eliminated since chemical potentials in any flavour will effectively impact neutron-proton equilibrium.

What does this mean?

LMA solar neutrino solution → close to complete flavour equilibration just before BBN, which sets the best limit on the lepton number of the universe:

Taking, *very conservatively*: $\xi_\mu < 0.3$

the new limit is: $\Delta N_\nu < 0.04$

HUGE improvement over the old limit: $\Delta N_\nu < \text{ a few}$

Dolgov et al.; Wong; Abazajian, Beacom & Bell.

Implication: no uncertainty on n in: $\rho_\nu = m_\nu n_\nu$

Summary

•Sterile neutrinos would require the existence of neutrino asymmetries in the early universe, in order to avoid BBN problems.

•The LMA (large mixing angle) solar neutrino solution
→equilibration of neutrino flavours just before weak freezeout

•The equilibration takes place via an MSW transition, synchronised across momentum modes due to neutrino-neutrino forward scattering.

•The stringent constraints on ν_e apply to all three flavours

•If a non-standard contribution to the relativistic energy density were to be detected, say, via the CMB, its origin would be something more exotic than neutrino degeneracy, since we have a very tight limit on the relic neutrino number density.